Many particle magnetic dipole–dipole and hydrodynamic interactions in magnetizable stent assisted magnetic drug targeting

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1. Introduction

The development of more effective drug treatment methodologies is an area of much research. In most drug delivery systems much of any drug administered to patients does not reach its target site. The aim of the drug targeting is to decrease the amount of drug delivered to healthy tissue, while maintaining the therapeutic action at the desired site. One such approach is magnetic drug targeting (MDT). For instance magnetic particles can be employed as carriers in a cancer treatment, thereby avoiding the side effects of conventional chemotherapy [1,2]. MDT can be employed as carriers in a cancer treatment, thereby avoiding the side effects of conventional chemotherapy [1,2].

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Fine Wire Co.) is taken as the wire material for the stent with a 62.5 μm radius following Avilés et al. [14]. The stent is prepared by looping a length of wire, L, into a 2 cm long coil having a 0.04 cm radius containing 10 loops, N, with 0.2 cm between each loop. Between use, each stent wire is cleaned by a 30 minute sonication in ethanol. A set of 15 identical coil stents are made and cleaned for the full MDT experimental testing.

The stent is firmly positioned within a borosilicate glass capillary tube by interference adhesion against the inner surface of the tube (radius of 0.04 cm). Controlled thickness capillary tubing is used to maximize the contrast between stent and glass curvature for real time video imaging and particle detection. Furthermore, this is also eliminates any turbulence caused by the irregular glass surface roughness. In this experiment we use a capillary glass tube (0.04 cm radius) and particle size proportionally similar to Avilés et al. [14].

The experimental setup is shown in Fig. 1. It consists of a capillary glass tube with a regularly spaced coil stent, an equally spaced capillary glass tube by interference adhesion against the inner surface of the tube (radius of 0.04 cm). Controlled thickness capillary tubing is used to maximize the contrast between stent and glass curvature for real time video imaging and particle detection. Furthermore, this is also eliminates any turbulence caused by the irregular glass surface roughness. In this experiment we use a capillary glass tube (0.04 cm radius) and particle size proportionally similar to Avilés et al. [14].

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The coil stent is then inserted into the tube and two homogeneous magnetic field strengths $\mu_0H_0 = 0.15$ T and $\mu_0H_0 = 0.60$ T are applied for different fluid velocities ranging between 0.58 cm/s and 52.6 cm/s. Once the magnetic field is applied the MDCPs were seen to agglomerate and create clusters. Different flow rates were chosen similar to those Avilés et al. [14]. For $\mu_0H_0 = 0.15$ T magnetic field strength 0.05, 0.1, 0.2, 0.4, 1.0 cm/s injection velocities and for $\mu_0H_0 = 0.60$ T magnetic field strength 0.2, 0.4, 1.0, 2.0, 4.5 cm/s injection velocities were used.

The amount of the MDCPs collected by the stent is measured by the differential between the MDCP concentration in the collection tube and the known initial particle concentration. Both solutions are measured by flow cytometry in triplicate counts.

After each particle solution injection the magnetic gradient was removed to demagnetize the superparamagnetic particles and to account for the mechanically bound particle residuals (always < 1% of the overall injected volume).

3. Outline of model

In order to effectively model this system, the 3D geometry of the stent and tube is reduced to 2D slice through the center of the tube (See Fig. 2). Thus the coiled stent is modeled as a series of circular cross sections of an infinite wire with radius of $R_{wire}$ located at the upper and lower boundaries of the walls. At each wall the wires are separated by a distance, $h$, between their centers, and the upper and lower sections are offset by $h/2$ as shown in Fig. 2. It should be noted that physically this
corresponds to a 2D description of flow with a parabolic profile in a rectangular box with transverse cylindrical wires, all of infinite extent. We model the behavior of \( N \) (\( N < 25 \)) MDCPs under the influence of Stokes drag, a force due to hydrodynamic interaction, and a magnetic force, modified to incorporate the mutual magnetic dipole–dipole interaction. Other forces such as inertia and gravity are ignored. The Stokes drag for MDCP \( n \) is

\[
\vec{F}_s = 6\pi\eta R_p (\vec{v}_f - \vec{v}_p),
\]

where \( \eta \) is the viscosity of the fluid, \( R_p \) is the radius of MDCP \( n \), and \( \vec{v}_f \) and \( \vec{v}_p \) are the velocities of the fluid and MDCP \( n \) respectively. The fluid velocity, \( \vec{v}_f \) is determined by solving the appropriate Navier–Stokes equations. The motion of a MDCP through a viscous fluid creates a disturbance to the fluid flow, which will be felt by all other MDCPs. As a result, the other MDCPs experience a force which is said to result from hydrodynamic interaction with the original MDCP. By considering \( N \) MDCPs, the force due to the hydrodynamic interaction, \( \vec{F}_{hyd} \), which acts on MDCP \( n \) due to the presence of other \((N-1)\) MDCPs, can be written as

\[
\vec{F}_{hyd} = \sum_{(i \neq n)} N \vec{z}_m \cdot (\vec{v}_i - \vec{v}_p),
\]

where \( \vec{z}_m \) is the modification due to the hydrodynamic interaction given by

\[
\vec{z}_m = -6\pi\eta R_p \frac{3R_p}{4|\vec{r}_{nI} - \vec{r}_i|} \left( 1 + \frac{(\vec{r}_{nI} - \vec{r}_i) \otimes (\vec{r}_{nI} - \vec{r}_i)}{|\vec{r}_{nI} - \vec{r}_i|^2} \right),
\]

where \( R_p \) is the radius of the MDCP \( i \), \( I \) is the unit tensor, \( \otimes \) is the vector tensor product (outer product), \( \vec{r}_n \) and \( \vec{r}_i \) are the positions of MDCP \( n \) and MDCP \( i \) respectively. Initially all MDCPs are taken to have the same radius but after agglomeration, MDCPs of different radius are possible, as each agglomeration is viewed as a new MDCP of increased radius.

In general the magnetic force acting on a magnetic moment is determined by

\[
\vec{F}_m = (\vec{m} \cdot \nabla)\vec{B}_{total},
\]

where \( \vec{m} \) is the magnetic moment and \( \vec{B}_{total} \) is the total magnetic flux density. Magnetic dipoles exert a force on each other, which can be included in the magnetic force equation by considering (i) the modified magnetic flux density and (ii) the modification in the magnetic moment resulting from this modified flux density. With regard to the magnetic dipole–dipole interaction between \( N \) number of MDCPs, each MDCP is taken as spherical with radius \( R_p \) and sufficiently small to have homogeneous magnetic flux throughout the MDCPs. Hence, in order to include the magnetic effect on MDCP \( n \) of the other \((N-1)\) MDCPs, the modified magnetic force, \( \vec{F}_{mmn} \), can be written as

\[
\vec{F}_{mmn} = (\vec{m}_n \cdot \nabla)\vec{B}_{total},
\]

where \( \vec{m}_n \) is the total magnetic moment of MDCP \( n \), \( \vec{B}_{total} \) is the total magnetic flux acting on MDCP \( n \). It can be taken as

\[
\vec{B}_{total} = \vec{B} + \sum_{(i \neq n)} N \vec{d} \vec{B}_i,
\]

where \( \vec{B} \) is the magnetic flux density due to the external field, \( d\vec{B}_i \) is the modification of the resulting magnetic flux density due to MDCP \( n \) at \( \vec{r} \). The modification to the magnetic flux density is thus taken as

\[
d\vec{B}_i(\vec{r}) = \frac{1}{3} \left( \mu_0 M_{m,p,s} \frac{L(\beta_i)}{B} \right) \vec{R}_{ri} \frac{1}{|\vec{r} - \vec{r}_i|^3} \left( 3(\vec{B}(\vec{r}_i) \cdot (\vec{r} - \vec{r}_i)) (\vec{r} - \vec{r}_i) - \vec{B}(\vec{r}_i) \right),
\]

where \( \mu_0 \) is the magnetic permeability of free space, \( \vec{r} \) represents an arbitrary point in space, \( \vec{B}(\vec{r}_i) \) is the flux density at \( \vec{r}_i \) and \( M_{m,p,s} \) is the saturation magnetization of the ferromagnetic material in the MDCP. The value of \( \vec{B} \) required to calculate the magnetic force as given by Eqs. (5) and (16), is calculated from the scalar magnetic potential due to the stent wires, which satisfies the Laplace equation over two conjoined regions: inside and outside the stent wires. Thus for outside the stent wires regions we have magnetic flux given by [9–15]

\[
\vec{B} = \mu_0(\vec{H}_0 - \nabla\phi),
\]

where \( \vec{H}_0 \) is the applied homogeneous magnetic field as in Fig. 2 and \( \phi \) represents the reduced magnetic scalar potential which in

Fig. 2. Schematic of the control volume (CV) used for determining the magnetizable stent collection efficiency (CE) through analysis of the corresponding MDCP trajectories. The CV has dimensions of 2 cm and 0.05 cm and encompasses a ten-loop stent within an expanded vessel. The MDCPs enter the CV from the left with a reduced average velocity defined by a parabolic profile and unexpanded average blood vessel velocity.
the region outside the stent wires is given by [20,19,21]

$$\phi = H_0 R_{wire}^2 \alpha_{wire} \frac{x \cos \theta + y \sin \theta}{x^2 + y^2},$$

(9)

where $R_{wire}$ is the radius of the stent wire implant, $\alpha_{wire}$ is the demagnetizing factor of the stent wire (given by Eq. (11)). The induced magnetization of the wire, $M_{wire}$, is taken to be parallel to the external magnetic field, $H_0$, and can be calculated from

$$\vec{M} = 2 \alpha_{wire} H_0,$$

(10)

where $\alpha_{wire}$ is the demagnetizing factor for an infinitely long cylinder in a perpendicular field taken as

$$\alpha_{wire} = \min \left( \frac{\mathcal{X}_{wire,0}}{2 + \mathcal{X}_{wire,0}}, \frac{M_{wire}}{2H_0} \right),$$

(11)

where $\mathcal{X}_{wire,0}$ and $M_{wire,s}$ are the zero field susceptibility and saturation magnetization of the ferromagnetic wire respectively and $H_0$ can be written

$$\vec{H}_0 = \left( \frac{H_0 \cos \theta}{H_0 \sin \theta} \right),$$

(12)

where $H_0$ is the magnitude of the applied field and $\theta$ is the direction of the applied magnetic field with respect to the x-axis, as in Fig. 2.

It is assumed that the ferromagnetic material in each MDCP consists of smaller single domain spherical nanoparticles. Thus, the average projection of $\vec{m}$ the moment in the direction of $\vec{B}_{\text{total}}$ can be calculated from the Langevin function [6,8,22–24]

$$L(\beta) = \coth(\beta) - \frac{1}{\beta},$$

(13)

with Langevin argument

$$\beta = \frac{m_{\text{fm,p}} \vec{B}_{\text{total}}}{kT},$$

(14)

where $\vec{B}_{\text{total}}$ is the magnitude of $\vec{B}_{\text{total}}$, $k$ is Boltzmann’s constant, $T$ is the absolute temperature and $m_{\text{fm,p}}$ is the magnitude of the magnetic moment of the magnetite in the MDCPs. The magnetic moment of each magnetite nanoparticle within the MDCP, $m_{\text{fm,p}}$, can be written as

$$m_{\text{fm,p}} = V_{\text{fm,p}} M_{\text{fm,p},s} \vec{B},$$

(15)

where $V_{\text{fm,p}}$ is the spherical volume of a single domain magnetite nanoparticle and $M_{\text{fm,p},s}$ is the (volume) saturation magnetization of the magnetite inside the MDCPs. Note that $M_{\text{fm,p},s}$ and $m_{\text{fm,p}}$ are fitting parameters in this model, obtained by Avilés et al. through characterization of the magnetic fluid [14].

Thus, the magnetic moment of the MDCP, $\vec{m}$, can be written as

$$\vec{m} = \omega_{\text{fm,p}} V_p m_{\text{fm,p}} L(\beta) \vec{B},$$

(16)

where $V_p$ is the MDCP volume and $\omega_{\text{fm,p}}$ is the volume fraction of ferromagnetic material in the MDCP, related to its weight fraction $\chi_{\text{fm,p}}$ through [9]

$$\omega_{\text{fm,p}} = \frac{\chi_{\text{fm,p}}}{\chi_{\text{fm,p}} + (1-\chi_{\text{fm,p}}) \rho_{\text{pol,p}} / \rho_{\text{pol,p}}},$$

(17)

where $\rho_{\text{pol,p}}$ is the density of the ferromagnetic material in the MDCP and $\rho_{\text{pol,p}}$ is the density of the polymer material in the MDCP. In this model the value of $\omega_{\text{fm,p}}$ is measured through the experiment.

4. Fluid flow—the Navier–Stokes equations

The fluid is treated as an incompressible, Newtonian, isothermal, single-phase fluid with velocity $\vec{v}_f$ and pressure $P$ at steady state flow. We have the continuity equation

$$\nabla \cdot \vec{v}_f = 0,$$

(18)

and the Navier–Stokes equation

$$\rho_f [ (\vec{v}_f \cdot \nabla) \vec{v}_f ] = \nabla P + \eta_f \nabla^2 \vec{v}_f,$$

(19)

where $\rho_f$ is the density of the fluid. To solve Eqs. (18) and (19), a parabolic velocity profile is assumed at the inlet control volume (CV) such that

$$v_{f,ax} = 1.5 u_0 \left( 1 - \frac{y}{R_{\text{vessel}}} \right)^2,$$

(20)

$$v_{f,yz} = 0,$$

(21)

where $u_0$ is the average inlet fluid velocity and $R_{\text{vessel}}$ is the vessel (tube) radius. Furthermore, non-slip boundary conditions ($\vec{v}_f = 0$) are applied at the wire-fluid interface and at the upper and lower CV boundaries. Atmospheric pressure is assumed at the outlet of the CV to satisfy the boundary condition on pressure.

5. Velocity equations, streamlines and capture cross section

The velocity of a MDCP $n$ can be obtained by summing the Stokes drag, the force due to hydrodynamic interaction and the modified magnetic force, as given in Eqs. (1), (2) and (5) respectively with inertial forces, $F_{\text{int}}$, as

$$F_n^\text{pol} + F_{\text{hyd}} + F_{\text{int}} = F_n^\text{pol}.$$

(22)

For MDCP $n$, by ignoring the inertial forces, $F_{\text{int}}$, we rewrite Eq. (22) as

$$6 \pi \eta R_p (\vec{v}_f - \vec{v}_p) + \sum_{i=1}^{N} (\vec{m}_i \cdot \nabla) \vec{B}_{\text{total}}|_{n} = 0.$$

(23)

Hence, we can obtain $\vec{v}_p$ by solving Eq. (23) numerically in each time step.

Finally, the trajectories of each MDCP can be obtained from evaluating the streamline functions [6,13]. The system performance of this mathematical model is calculated in terms of collection efficiency, CE, defined as

$$CE = \frac{2 R_{\text{vessel}} (y_1 + y_2)}{2 R_{\text{vessel}}} 100,$$

(24)

where $y_1$ and $y_2$ are defined by the location of the streamline at the entrance to the CV of the last MDCPs captured by the stent wires (Fig. 2). All calculations were performed using the open-source software finite volume library OpenFOAM [25].

6. Results and discussions

In this paper, we include the effect of both magnetic dipole–dipole and hydrodynamic interactions for multiple MDCPs in the stent based mathematical model of Avilés et al. [14]. We focus on varying the initial positions of $N$ ($N < 25$) MDCPs at the entrance of the CV and present the results in terms of the CE of the system considering the agglomeration of MDCPs.

Of interest is the effect of the velocity of the blood and the field strength on the CE of the system. This is shown in Figs. 3–6 with both dipole-dipole magnetic and hydrodynamic interactions, experimental results and without any particle interactions.
The collection efficiency (CE) of the system plotted as a function of the blood velocity (2.1, 4.2, 10.6, 21.2, 42.4 cm/s) at the applied field $\mu_0H_0 = 0.65T$.

In the 2D model, the behavior of the MDCPs after agglomeration is also considered. It is seen that the MDCPs create a cluster during their agglomeration as a result of both interactions. The volume of the cluster is calculated by summing the volume of the MDCPs agglomerated and the radius of the cluster is calculated using the general volume formulation $\left(\frac{4}{3}\pi r^3\right)$ [26]. Whilst this assumption does not account fully for the resulting hydrodynamic volume, the effect of this assumption should not significantly affect our results.

6.1. Mathematical model explanation and details

The rationale for the simulations is as follows. Given sufficient computing power, one might consider randomly distributing, particle in the form of a cluster, a very large number ($>10,000$) of MDCPs and allow interactions between all of these. With limited computing resources, one is forced to reduce this. We do this in two ways. Firstly, by limiting the regions of initial positions that we consider and secondly by limiting the number of MDCPs that we allow to mutually interact. Thus we consider only those parts of the simulation which are likely to contribute to any alteration in the CE. For instance, in those parts of the capture cross section closest to the vessel walls, one can expect no improvement in the CE. In fact it is only where the initial positions are close to the border between the collection and no collection region, that is around the boundary of the reference capture cross section that we start to see altered trajectories due to interactions. The boundary of the reference capture cross section (CCS), $\xi_c$ is the trajectory of the last MDCP, which would be captured by the stent wires in the non-interacting case. Secondly, the mutual inter-particle interaction would not be expected to have infinite extent. One can postulate a number of MDCPs in the model where the predicted difference in performance between modeling $N$ and $N' + 1$ becomes arbitrarily small. We point out that the computational effort required to model interactions scales with $N^2$, where $N$ is the number of MDCPs interacting. Simulations were performed for increasing $N$, and the results indicate that there is no significant change to the system performance metrics beyond twenty five MDCPs.

In light of these factors, we consider a particular, homogeneously distributed cluster of $N$ MDCPs. The MDCP concentration of the Avilès et al. system is 50 mg/l which corresponds to $11.2 \times 10^{10}$ MDCPs per liter and the MDCP concentration of our experimental setup is $4 \times 10^{10}$ per liter. The effective initial
distance between the MDCPs in the CV is calculated using the concentration of the MDCPs in the glass tube. Initial distance is taken as the cube root of the MDCPs amount per liter ((dm)³) and we created a homogeneous rectangular cluster of particles which mimic the experimental particle concentration flowing through the stent during the video streaming.

In order to describe the effect of both interactions we consider two different simulation configurations, similar to those used in a previous paper for the inclusion of interactions between the two MDCPs and between the MDCPs and the fluid [19]. The first configuration is intended to illustrate the agglomeration of the MDCPs within the reference CCS region. In this configuration all of the MDCPs are captured, as expected and the resulting CE of the system for this situation is unaltered.

The second simulation configuration is intended to examine the effects of interactions on the CE of the system near the \( \lambda_c \). For this, we place the center of the particle cluster on the \( \lambda_c \) for a given velocity and record changes in CE through following the MDCP trajectories in the normal way. We then shift the particle cluster up and down, and again record changes in CE. This approach is repeated for each increased fluid velocity, using, for a given field, the same particle cluster.

6.2. Comparison of the mathematical model results and literature

Initially, the results of our mathematical model and the experimental result of Avilés et al. are compared. Results are presented by calculating the CEs for identical MDCPs with initial radius \( \text{R}_p = 0.435 \mu m \) containing 25 wt% magnetite, under the influence of homogeneous magnetic field oriented perpendicularly to the flow (\( \theta = \pi/2 \)) with magnitudes of 0.17–0.65 T. The glass tube radius size is taken as 0.05 cm as in the experiment of Avilés et al. In the model the magnetization of the individual MDCPs is taken as the average value given by the Langevin function due to the single domain magnetic nanoparticles within. The relevant fluid flow properties and the properties of the ferromagnetic materials used in the MDCPs and for the stent wire, are given in Table 1.

For the configurations outlined above, we keep the applied field constant (\( \text{H}_0 = 0.17 \text{T} \)) and we increase the blood velocity from \( u_0 = 2.1 \) to 42.4 cm/s. The resulting CEs for these simulations are shown in Fig. 3. Secondly, using the same methodology we applied \( \text{H}_0 = 0.65 \text{T} \) and vary the fluid velocity between \( u_0 = 2.1 \) and 42.4 cm/s. The resulting CEs are given in Fig. 4.

Table 1
Experimental values of fluidic setup and material parameters used in the mathematical model of the stent based simulation.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Units</th>
<th>Values</th>
<th>Data type</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MDCPs properties</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Polymer material</td>
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<td></td>
<td>P(3(V-COOH))Mag</td>
<td>Physical</td>
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<tr>
<td>Radius</td>
<td></td>
<td>( \mu m )</td>
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<td>Physical</td>
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<tr>
<td>Saturation magnetization</td>
<td>( M_{\text{Ms}} )</td>
<td>kA/m</td>
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<td>Measured</td>
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<td><strong>MDCPs magnetic material properties</strong></td>
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<tr>
<td>Material</td>
<td></td>
<td></td>
<td>Magnetite</td>
<td>Physical</td>
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<tr>
<td>Weight content</td>
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<td>Volume content</td>
<td>( \omega_{\text{MnP}} )</td>
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<td>6.4</td>
<td>Measured</td>
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<td>Saturation magnetization</td>
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<td>K</td>
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<td>Boltzmann’s constant</td>
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<td>Permeability of vacuum</td>
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<td>Tm/A</td>
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<td><strong>Applied field properties</strong></td>
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<tr>
<td>Magnitude</td>
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<td>T</td>
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<td>Angle of field direction</td>
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<td>( \pi/2 )</td>
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<td><strong>Stent properties</strong></td>
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<td>cm</td>
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<tr>
<td>Number of loops</td>
<td>( N_l )</td>
<td>–</td>
<td>10</td>
<td>Physical</td>
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<tr>
<td>Coil length</td>
<td>( L )</td>
<td>cm</td>
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<td>Physical</td>
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<td>1000</td>
<td>Physical</td>
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<td><strong>Blood &amp; vessel properties</strong></td>
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<tr>
<td>Velocity</td>
<td>( u_0 )</td>
<td>cm/s</td>
<td>2.1, 4.2, 10.6, 21.2, 42.4</td>
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</tr>
<tr>
<td>Volume</td>
<td>( V_{\text{blood}} )</td>
<td>cm³</td>
<td>0.58, 1.17, 2.34, 4.68, 11.7, 23.4, 52.6</td>
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<tr>
<td>Density</td>
<td>( \rho )</td>
<td>kg/cm³</td>
<td>1000</td>
<td>Physical</td>
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<tr>
<td>Viscosity</td>
<td>( \eta )</td>
<td>kg/ms</td>
<td>( 1.0 \times 10^{-3} )</td>
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<tr>
<td>Vessel radius</td>
<td>( R_{\text{vessel}} )</td>
<td>cm</td>
<td>0.05, 0.04</td>
<td>Physical</td>
</tr>
</tbody>
</table>

Bold values are used in our experiment. Some material parameters are in agreement with Avilés and coworker study [14].
In Figs. 3 and 4, the results of the mathematical model with the interactions show closer agreement with experimental results of Avilés et al. with low fluid velocity. This is due to the interaction and agglomeration of MDCPs in our model. With low fluid velocity (< 10 cm/s) and higher applied field ($\mu_0 H_0 = 0.65$ T) MDCPs create a larger volume of cluster more easily than with the lower applied field ($\mu_0 H_0 = 0.17$ T). When we increase the fluid velocity the likelihood of the agglomeration of the MDCPs starts to decrease. For higher fluid velocity the CE of the IA-MDT system predicts lower collection than the results of Avilés model without interactions. This is due to the effect of hydrodynamic interactions on the velocity of MDCPs and so the trajectories of the MDCPs.

### 6.3. Comparison of the mathematical model and experimental results

Next, we compare the results of the mathematical model and in vitro experiments undertaken at CRANN TCD. Results are presented by calculating the CEs for identical MDCPs with initial length. $R_p = 0.43 \mu m$ containing 45.8 wt% magnetic, under the influence of homogeneous magnetic field oriented perpendicularly to the flow ($\theta = \pi/2$) with magnitudes of 0.15 T and 0.60 T. The glass tube radius size is 0.04 cm in our experiments. This was done to achieve a better image contrast between the particle layers aggregating on the stent during the experimental testing which is also increased by the smaller capillary diameter when compared to Avilés et al. [14] model.

In the model the magnetization of the individual MDCPs is taken as the average value given by the Langevin function due to the single domain magnetic nanoparticles within. The relevant fluid flow properties and the properties of the ferromagnetic material used in the MDCP and for the stent wire, are given in Table 1.

For the configurations outlined above, we keep the applied field constant ($\mu_0 H_0 = 0.15$ T) and we increase the blood velocity up to $u_0 = 11.7$ cm/s. The resulting CEs for these simulations are shown in Fig. 5. Secondly, we apply $\mu_0 H_0 = 0.60$ T and increase the fluid velocity up to $u_0 = 52.6$ cm/s. The resulting CEs are given in Fig. 6. In Figs. 5 and 6, the results of the model with the interactions show closer agreement with the measured experimental results. The results shown also highlight how a 0.01 cm reduction in the capillary radius can affect the collection efficiency. This leads to speculation over a higher efficacy of the MDCT technique at the level of peripheral circulatory capillary vessels. On the other hand, this increased CE efficiency also increases the risk of vessels clotting and thrombolytic effect especially when also accounting for the presence of the solid part of the blood [27].

Collection Efficiency is a key parameter for the modeling validation of the experimental testing. Differences between Avilés et al. and our experimental model (Cregg et al.) are shown in Table 2.

### 6.2. Experimental results

In Figs. 5 and 6, the results of the model with the interactions show closer agreement with experimental results of Avilés et al. with low fluid velocity. This is due to the interaction and agglomeration of MDCPs in our model. With low fluid velocity (< 10 cm/s) and higher applied field ($\mu_0 H_0 = 0.65$ T) MDCPs create a larger volume of cluster more easily than with the lower applied field ($\mu_0 H_0 = 0.17$ T). When we increase the fluid velocity the likelihood of the agglomeration of the MDCPs starts to decrease. For higher fluid velocity the CE of the IA-MDT system predicts lower collection than the results of Avilés model without interactions. This is due to the effect of hydrodynamic interactions on the velocity of MDCPs and so the trajectories of the MDCPs.

### 7. Conclusions

We have presented an interaction model applied to IA-MTD. This model considered the agglomeration of particles known to occur in such systems [12,14,15]. We include the effects of both the dipole-dipole and hydrodynamic interactions for multiple particles in stent implant arrangements. The resulting collection efficiencies derived from the mathematical model are in closer agreement with our latest experimental results and those presented by Avilés et al. Furthermore, the mathematical model presented in this work represents a useful analytical tool for the prediction of the efficacy of targeted drug delivery by super-paramagnetic particles. The implications in the nanotechnology and nanomedicine research area are based on the efficiency in delivering the drug coated particles within the magnetizable stent length.

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### References


